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Analysis of the particle-wall interaction in a granular layer of repelling magnetic particles

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Abstract

In this work we aimed to describe more accurately the force of interaction between the magnetic dipoles and the magnetic walls in a granular layer of repelling magnetic particles confined in a Hele-Shaw cell. We show, through an analytical expression evaluated numerically, that, in the system we are interested in, the walls can be modelled as being virtually infinite, in numerical terms.

Keywords: compression dynamics; granular damper; magnets

Introduction

In [1], the compression-expansion dynamics of a granular bed composed of cylindrical repelling magnets of diameter D and magnetic moment intensity μ contained in a two-dimensional Hele-Shaw cell is studied. This is a toy model for the compression of a granular bed, since we have only repulsive and friction forces, but no particle-particle contact forces. Nevertheless, this toy model is of interest in itself, because it can be understood as the basis for the development of magnetic dampers, for example.

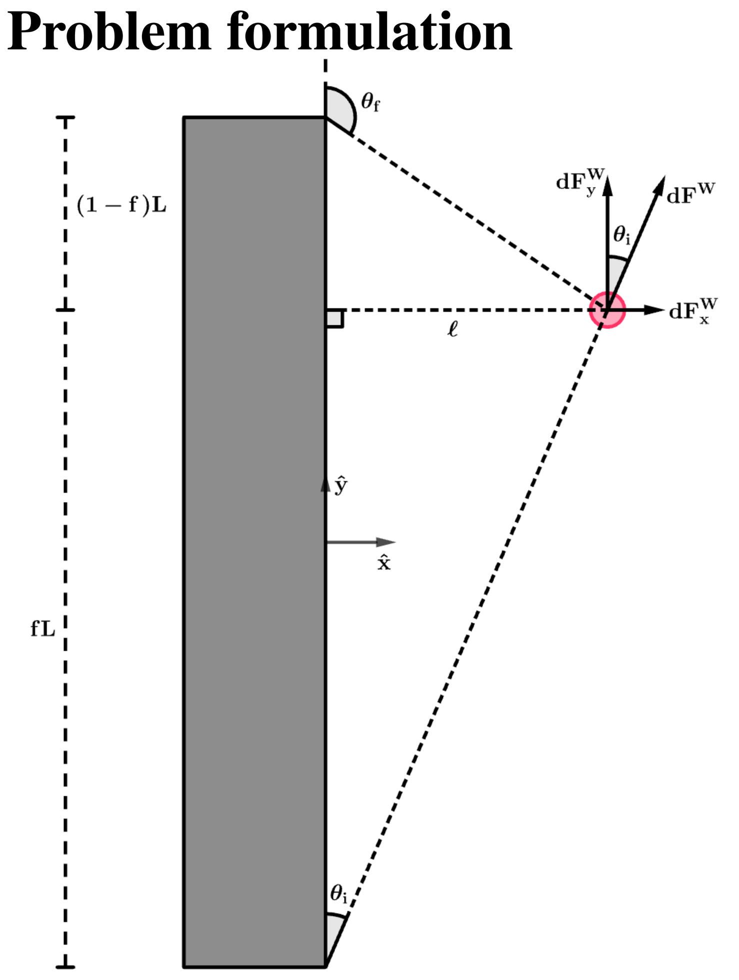
Objectives

In this work, our goal was to improve, quantitatively, the agreement between numerical simulation and experiment data of the system explored in [1], with an improvement of the description of the walls of the Hele-Shaw cell.

Theoretical background

To achieve the goal established, we considered the expressions for the magnetic force and torque between two magnetic dipoles, given respectively in [2] and [3].

Problem formulation



In [1] the authors verified experimentally that the particle-wall interaction force, F_{pw}, should be proportional to $\ell^{-3.5}$, with ℓ being the perpendicular distance of the particle to the wall. With the mathematical model considered in Section 4.2 of [1] and considering, instead, a *finite* wall of length L and magnetic moment per unit length M, we have that the point dipole feels a magnetic repulsion force element given by

$$dF_x^W = -\frac{3\mu_0 M\mu}{4\pi\ell^3}\sin^3\theta\, d\theta, \tag{1}$$

where μ_0 is the magnetic permeability of vacuum.

Integrating dF_x^W with respect to θ and substituting $M = \mu/D$, we obtain

$$F_x^W = -\frac{\mathcal{J}}{D\ell^3}\left(\frac{\cos^3\theta}{3} - \cos\theta\right)\Bigg|_{\theta_i}^{\theta_f} = -\frac{\mathcal{J}}{D\ell^3}\left[\frac{1}{3}\cos^3(\theta_f) - \cos(\theta_f) - \frac{1}{3}\cos^3(\theta_i) + \cos(\theta_i)\right], \tag{2}$$

with $\mathcal{J} = 3\mu_0\mu^2/4\pi$ and

$$\theta_i = \arctan\left(\frac{\ell}{fL}\right) = \arctan A, \tag{3}$$

$$\theta_f = \pi - \arctan\left(\frac{\ell}{(1-f)L}\right) = \pi - \arctan B, \tag{4}$$

considering, for simplicity, $\arctan(+\infty) = \pi/2$. We can then simplify equation (2) using

$$\cos(\arctan x) = \frac{1}{\sqrt{x^2+1}}, x \in (-\pi/2, \pi/2), \tag{5}$$

and equations (3) and (4) to obtain:

$$F_x^W = -\frac{\mathcal{J}}{D\ell^3}\left[\frac{1}{\sqrt{A^2+1}}\left(\frac{3A^2+2}{3A^2+3}\right) + \frac{1}{\sqrt{B^2+1}}\left(\frac{3B^2+2}{3B^2+3}\right)\right]. \tag{6}$$

Numerical results

We plotted F_x^W $vs.$ ℓ in a log-log plot for fixed $L = 100D$ and several values of f, ignoring the constant factor $\mathcal{J}/D$. We also plotted the experimental dependence obtained in [1], still ignoring the corresponding constant $\mathcal{J}/D$. The resulting plots are shown below.

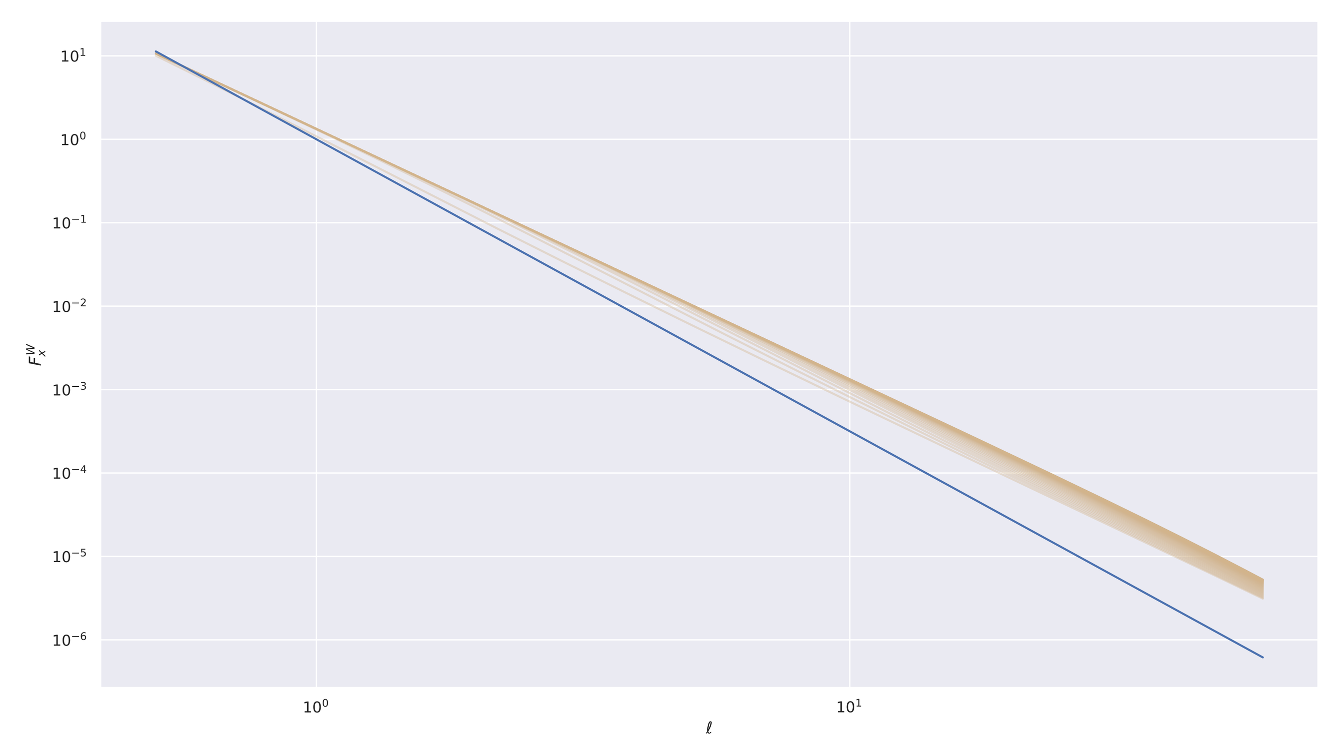


Figure 1: Log-log plot of the repulsion force versus ℓ. In blue we see the experimental fit expected.

This shows that $F_x^W \propto \ell^\alpha, \alpha < 0$, and also that α does not vary much for different values of f, as observed in the tan sheaf of lines; these lines were obtained for f varying from 0.005 to 0.995 with a step of 0.01 and ℓ in the range of 0.5 to 59 with a step of 0.01 as well, with a fit of the form $y = ax^b$. We also remark that the average of the values of α is ≈ -3.005.

Conclusions

Figure 1 shows that, although the wall is finite, we should not be too far out in modelling it as infinite, since for the infinite wall we have $F_x^W \propto \ell^{-3}$, with the same constant of proportionality. In particular, the expected proportionality, $F_x^W \propto \ell^{-3.5}$, is not recovered, which indicates that the mathematical model proposed in [1] for the particle-wall interaction is not capturing correctly the expected dependence on ℓ. Currently, we are trying to model the walls as finite magnetic bars and use the analytical expressions for the magnetic field to determine the particle-wall interaction. This approach also allows us to take into account field variations near the edges.

References

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